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V-shaped stable nonspiral patterns

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V-shaped stable patterns moving with constant velocity and shape are reproduced experimentally using the Belousov-Zhabotinsky reaction. In the literature, planar and rotating spiral waves are the only stable autonomous wave patterns whose shape remains constant during propagation. It was found that the velocity of the V-shaped waves increases by a factor up to 10 times the planar wave front velocity for the same excitable medium depending on the initial inner angle between asymptotes to the front. Moreover, these structures were found to be stable with respect to small perturbation of their shapes. Results are compared with theoretical predictions showing a perfect agreement.

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Theoretical predictions [1] have recently shown that V-shaped stable nonspiral autowave patterns [2] can exist in an unbounded excitable medium. These waves propagate through the medium with constant shape and velocity. Experimentally, only rotating spiral waves and planar waves [3–6] have been described in the literature as stable autowave patterns [7] with constant shape. Here, we reproduce experimentally the V-shaped patterns, using the Belousov-Zhabotinsky (BZ) reaction as the excitable medium [8].

Experiments were performed in a liquid BZ reaction [9] at room temperature (24 $\pm 1\,^\circ$C) with the following recipe: 0.08*M* malonic acid, 0.22*M* sulfuric acid, 0.28*M* sodium bromate, and 0.025*M* ferroin. The thickness of the liquid layer never surpassed 1.5 mm in order to avoid three-dimensional effects due to convection [9], and the Petri dish (9 cm in diameter) was covered to avoid any interaction between the reaction and the oxygen in the air.

The initial V-shaped wave front was obtained by means of a silver wire (10 cm long) [10] with appropriate shape immersed into the liquid. The free ends of the wire were situated as close as possible to the boundaries of the Petri dish to avoid spiral wave formation or other spurious effects. Different initial inner angles α were fixed before the experiments.

The experiment was followed with a charge coupled device (CCD) camera connected to a video recorder system. Single frames of the resulting movies were digitized by an image-acquisition card and finally analyzed on a personal computer. Then, wave velocity was measured in the direction of wave propagation by recording automatically the position of the vertex at constant intervals of time.

Once the medium had become homogeneous, the silver wire constantly emitted V-shaped waves with a period not greater than the refractoriness period supported by that medium [10]. The initial pattern evolves in time to its final configuration, with a constant inner angle α (Fig. 1), with a small relaxation time as predicted by the theory [1]. Small perturbations of the front shape [near the silver wire in Fig. 1(a)] decrease very quickly and the stable V-shaped pattern moves several centimeters without visible changes in its shape. We have also numerically investigated the V-shaped wave propagation on a standard model of an excitable medium (two variable Oregonator model [11–13]). Figure 1(b) shows a good agreement with the experimental results [Fig. 1(a)].

In our experiments, the silver wire vertex was placed far enough (≈ 4 cm) from the unpenetrable boundary of the Petri dish. We have not observed any influence of the Petri dish border on the motion of the V-shaped patterns; they remained stable and moving with constant velocity till they approached the boundary where annihilation of the wave fronts took place. That is why we can confirm that the observed V-shaped patterns, undoubtedly, will exist in unbounded medium, as it follows from the theory.

The properties of the obtained stable V-shaped patterns are in good agreement with the theoretical predictions. For nearly all the front length, its curvature remained equal to zero except at the vertex where a high negative curvature (measured in the direction of propagation) was observed [Figs. 1(c) and 1(d)]. Figure 2 shows the theoretical pattern [1] obtained in the framework of the kinematical theory [14] given by the natural equation

$$k(l) = -\frac{1}{D}\,\frac{V^2(0)-V_0^2}{V_0+V(0)\cosh\left(\dfrac{[V^2(0)-V_0^2]^{1/2}}{D}\,l\right)}, \qquad (1)$$

where $V(0)$ is the propagation velocity of the stationary structure moving as a whole along the direction of propagation, V_0 is the planar wave front velocity supported by the medium, D is the diffusion coefficient, and k is the wave front curvature depending on l (length of the wave front measured from the vertex).

The experimental wave velocities measured for different initial inner angles α proved to be in very good agreement with the theoretical prediction (Fig. 3)

$$V_0 = V(0)\sin(\alpha/2). \qquad (2)$$

Since the kinematical description given by Eqs. (1) and (2) assumes that only one wave front propagates through the medium, in Fig. 3 we have represented only the wave veloci-

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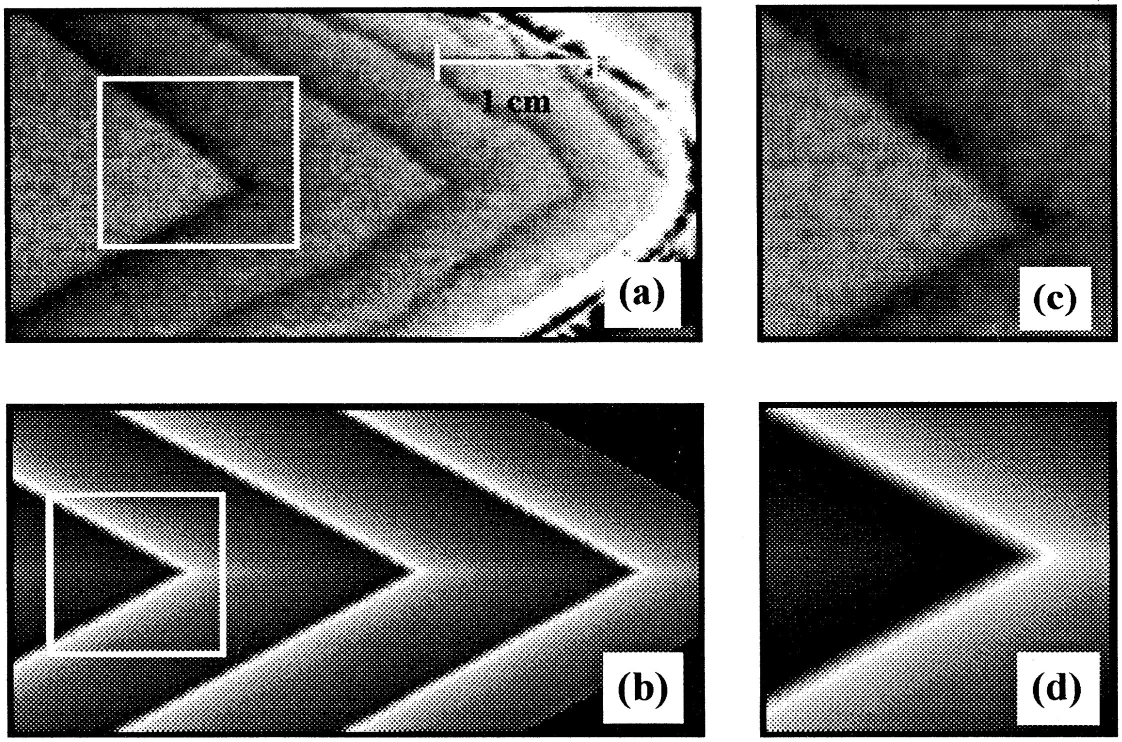

FIG. 1. V-shaped patterns moving to the left (a) obtained in the BZ reaction by means of a silver wire (its shadow is seen in the right side of the picture) constantly emitting waves and (b) for the Oregonator reaction-diffusion model (parameters: $f=3$, $\varepsilon=0.02$, $q=0.002$, and $D_u=1$). Details of the previous images, (c) for the BZ reaction and (d) for the Oregonator model, show the wave front shape near the vertex and the perfect agreement between numerics and experiments in this particular case. Initial inner angle α between the asymptotes to the fronts in both cases is nearly equal to 60°. Note that those initial instabilities in the wave front shape observed near the silver wire in (a) are damped towards the final stable V-shaped pattern. Note also that only near the vertex is the curvature of the wave extremely high.

ties measured for the first wave front emitted by the silver wire into the fresh medium. Smaller values of the velocity were observed for the successive wave fronts but they remained always close to the first one.

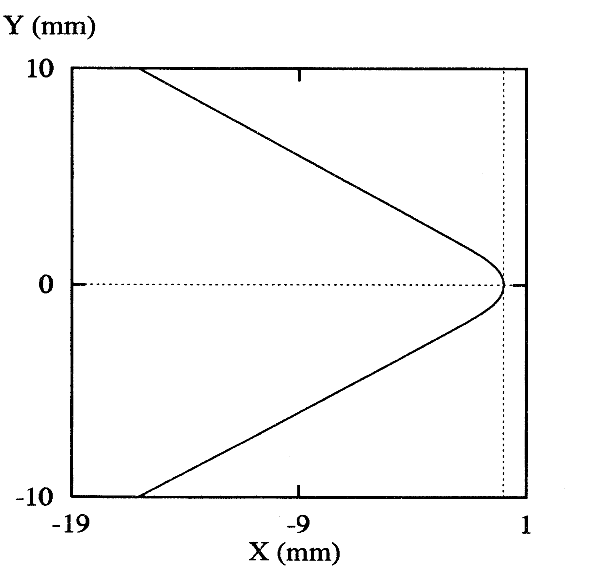

FIG. 2. Theoretical V-shaped pattern described by Eq. (1). Note the extremely high curvature near the vertex. Parameters D and V_0 are taken as those in the BZ reaction: $D=10^{-5}$ cm^2/s and $V_0=1$ mm/min. $\alpha=60°$.

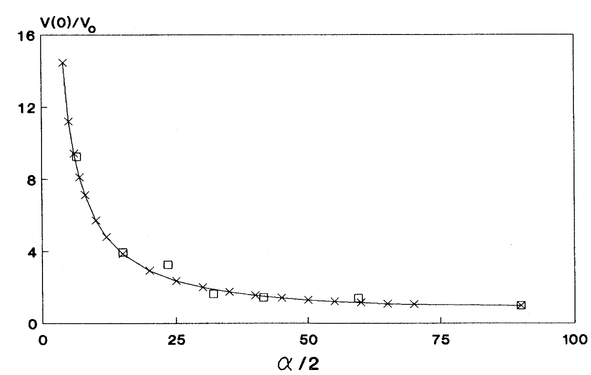

FIG. 3. Dependence of the normalized wave propagation velocity of the stationary structure $V(0)/V_0$ as a whole with the inner half-angle $\alpha/2$ between asymptotic fronts. Squares (□) correspond to measurements in the BZ reaction for the first emitted wave into the fresh medium, crosses (×) correspond to the numerical simulations of the Oregonator model, and the solid line corresponds to the theoretical prediction of Eq. (2).

The results of this paper show that, in a perfect agreement among theory, numerical simulations, and experiments, in homogeneous unbounded media there exists a family of nonspiral patterns with invariable shape of the front. These patterns are stable and move as a whole along a straight line. Their velocity may exceed essentially the velocity of a planar wave front by a factor depending on α.

Note that structures of invariable shape have been recently observed in an excitable medium with penetrable boundaries [15]. These patterns, however, have positive curvature near the vertex and a presence of penetrable boundaries is necessary for their existence.

The detailed study of these patterns should promote others to obtain new information on the autowave behavior in excitable media and in particular on the dependence of the velocity on the curvature (in the case of very large negative curvatures).

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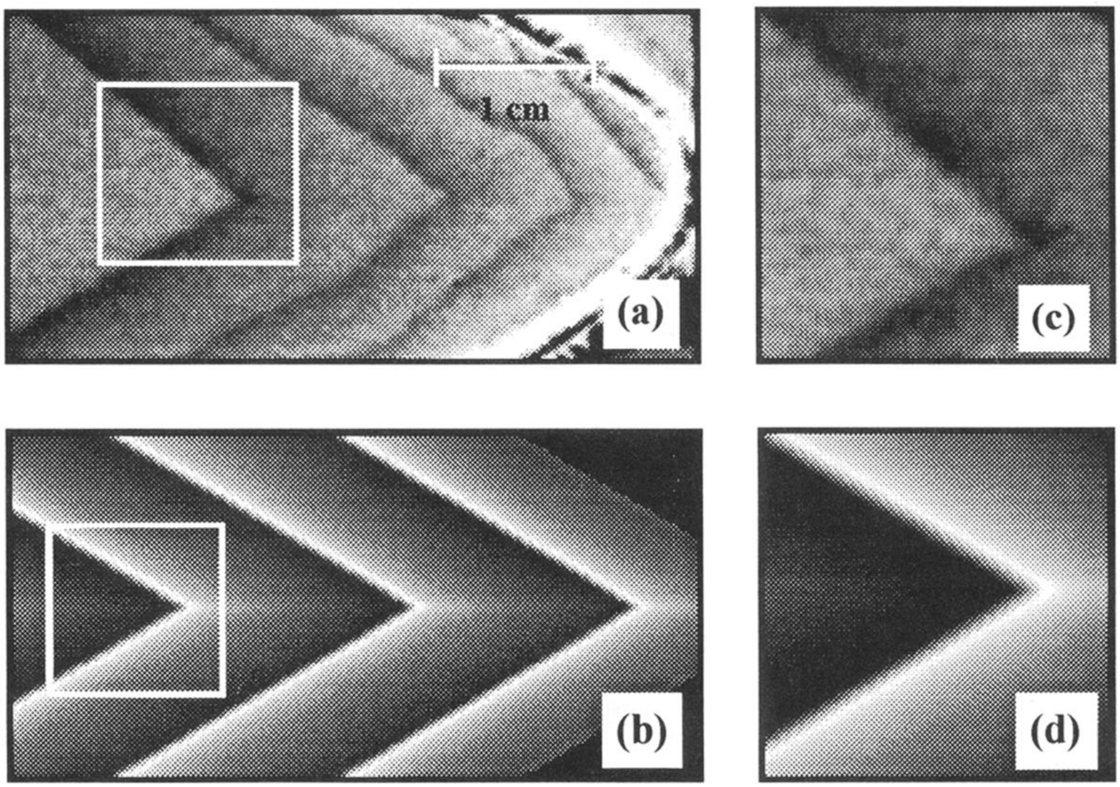
[1] P. K. Brazhnik and V. A. Davydov, Phys Lett. A (to be published).
[2] The term "autowaves" was coined by R. V. Khokhlov in *Autowave Processes in Systems with Diffusion*, edited by M. T. Grekhova (Gorky Academy of Sciences, USSR, 1981), Preface, as an abbreviation for "autonomous waves" since such waves can propagate without a forcing function. Autowaves are characteristic of strongly nonlinear active media and most of their properties can be found in V. I. Krinsky, in *Self-Organization: Autowaves and Structures Far From Equilibrium*, edited by V. I. Krinsky (Springer-Verlag, New York, 1984), pp. 9–19.
[3] A. T. Winfree, Sci. Am. **230**, 82 (1974).
[4] K. I. Agladze, V. I. Krinsky, and A. M. Pertsov, Nature (London) **308**, 834 (1984).
[5] S. C. Müller, T. Plesser, and B. Hess, Physica D **24**, 71 (1987).
[6] V. Pérez-Muñuzuri, R. Aliev, B. Vasiev, V. Pérez-Villar, and V. I. Krinsky, Nature (London) **353**, 740 (1991).
[7] *New Trends in Pattern Formation in Active Nonlinear Media*, edited by V. Pérez-Villar, V. Pérez-Muñuzuri, C. Pérez-García, and V. I. Krinsky, special issue of Int. J. Bif. Chaos **4** (5) (1994).
[8] A. N. Zaikin and A. M. Zhabotinsky, Nature (London) **225**, 535 (1970).
[9] J. Rodríguez and C. Vidal, J. Phys. Chem. **93**, 2737 (1989).
[10] K. Showalter, R. M. Noyes, and H. Turner, J. Am. Chem. Soc. **101**, 7463 (1979).
[11] R. J. Field, E. Köros, and R. M. Noyes, J. Am. Chem. Soc. **94**, 8649 (1972).
[12] W. Jahnke and A. T. Winfree, Int. J. Bif. Chaos **1**, 445 (1991).
[13] J. L. F. Porteiro, V. Pérez-Muñuzuri, and A. P. Muñuzuri, and V. Pérez-Villar, Physica D **69**, 309 (1993).
[14] A. S. Mikhailov, V. A. Davydov, and V. S. Zykov, Physica D **70**, 1 (1994).
[15] O. Steinbock, V. S. Zykov, and S. C. Müller, Phys. Rev. E **48**, 3295 (1993).



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